



Enhanced Nonlinear Estimation with Unscented Kalman Filter and RBF Neural Networks

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Abstract

This paper introduces an enhanced Unscented Kalman Filter (UKF) algorithm integrated with Radial Basis Function (RBF) neural networks to advance the accuracy of nonlinear state estimation in dynamic systems. Our approach specifically addresses the estimation of the State of Charge (SOC) of a battery, leveraging a second-order equivalent circuit model to capture the battery's complex behavior. The innovation of our method lies in the integration of RBF neural networks into the UKF framework, which enhances the algorithm's capability to model nonlinearities and improve prediction accuracy. The standard UKF algorithm, while robust in handling nonlinear systems, often struggles with certain nonlinearities inherent in battery SOC estimation. By incorporating an RBF neural network, which excels at approximating complex, nonlinear relationships, our proposed UKF-RBF algorithm achieves superior performance. The RBF network is trained to capture the nonlinear Open Circuit Voltage (OCV) vs. SOC relationship, which is crucial for accurate SOC estimation. Experimental results demonstrate that the UKF-RBF algorithm significantly outperforms the traditional UKF in terms of Mean Squared Error (MSE), Root Mean Square Error (RMSE), and Mean Absolute Error (MAE). The UKF-RBF algorithm shows marked improvements in SOC estimation accuracy across varying operating conditions and temperatures, making it a robust solution for practical applications in battery management systems. The integration of RBF neural networks into the UKF framework represents a novel approach that bridges the gap between traditional Kalman filtering and modern neural network techniques, providing a substantial enhancement in the estimation of nonlinear states.

Keywords:

Neural Networks, CNN, UKF-RBF, OCV

1.Introduction

Accurate estimation of the State of Charge (SOC) is pivotal for effective battery management systems, particularly in applications such as electric vehicles and renewable energy storage where battery performance directly impacts operational efficiency and safety. Traditional SOC estimation techniques, such as those relying on linear Kalman Filters, often struggle to account for the complex, nonlinear behavior inherent in battery systems, leading to less reliable predictions. The Unscented Kalman Filter (UKF) has been widely utilized for handling such nonlinearities due to its capability to approximate the state and observation models more accurately than linear filters. However, the performance of UKF can still be compromised by limitations in the modeling of process and observation noise. To overcome these challenges, this paper introduces an enhanced UKF algorithm that integrates Radial Basis Function (RBF) neural networks. The RBF network is employed to model the intricate nonlinear relationships between SOC, battery voltage, and current more precisely. By incorporating RBF into the UKF framework, our approach improves the accuracy and robustness of SOC estimation, as evidenced by comparative experiments. The enhanced UKF-RBF algorithm demonstrates superior performance over traditional UKF methods, providing a more reliable and precise estimation of SOC, which is crucial for advancing battery management technologies.

2.Methodology

This section provides a comprehensive description of the methodology used to enhance SOC estimation accuracy by integrating the Unscented Kalman Filter (UKF) with Radial Basis Function (RBF) neural networks. The methodology encompasses the battery model, data preparation, UKF implementation, RBF neural network training, and the combined UKF-RBF algorithm.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \exp\left(\frac{-T}{r1*c1}\right) & 0 \\ 0 & 0 & \exp\left(\frac{-T}{r2*c2}\right) \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{-T}{q} \\ r1\left(1 - \exp\left(-\frac{T}{r1*c1}\right)\right) \\ r2\left(1 - \exp\left(-\frac{T}{r2*c2}\right)\right) \end{bmatrix}$$

where T is the sampling time, q is the battery capacity in Coulombs, and r1, r2, c1, and c2 are model parameters.

2.1 Equivalent Circuit Model

The battery model used for SOC estimation is the second-order Equivalent Circuit Model (ECM). This model captures the battery's dynamic behavior using electrical components, which include resistors and capacitors.

2.1.1 Model Description

The ECM consists of a series resistance (r_0) and two parallel RC (resistor-capacitor) networks. This model effectively represents the battery's internal resistance and transient response.

2.1.2 Parameters

The parameters used in the ECM are derived from experimental data and are crucial for accurately simulating the battery's behavior. The key parameters are:

Rated capacity: 70 Ah

Actual capacity: 68.27 Ah

Series resistance (r_0): 0.0016270868 Ω

RC network parameters:

- $r_1 = 0.000062505 \Omega$
- $c_1 = 21126.57803 \text{ F}$
- $r_2 = 0.000354013 \Omega$
- $c_2 = 89368.53265 \text{ F}$

2.2 Data Preparation

Accurate SOC estimation requires high-quality data. This involves preparing the input data, including current and voltage measurements, and deriving the Open Circuit Voltage (OCV) vs. SOC relationship.

2.2.1 UKF Initialization

The UKF algorithm is initialized with the following parameters:

Sampling time: 0.1 seconds

Battery capacity: 68.27Ah (converted to Coulombs)

Initial state vector: [1, 0, 0]

Initial covariance matrix: $0.01 * \text{eye}(3)$

Process noise variance: $Q_UKF = 1e-6 * \text{diag}([1, 0.98, 0.52])$

Observation noise variance: $R_UKF = 0.05$

2.2.2 UKF-RBF Initialization

The UKF-RBF algorithm incorporates an RBF neural network to enhance the state prediction accuracy:

- Initial state vector: $[1, 0, 0]$
- Initial covariance matrix: $0.1 * \text{eye}(3)$
- Process noise variance: $Q_RBF = 1e-4 * \text{diag}([1, 0.98, 0.52])$
- Observation noise variance: $R_RBF = 0.1$
- RBF neural network parameters: goal = 0.01, spread = 1, max_neurons = 50

2.2.3 Battery pack test bench

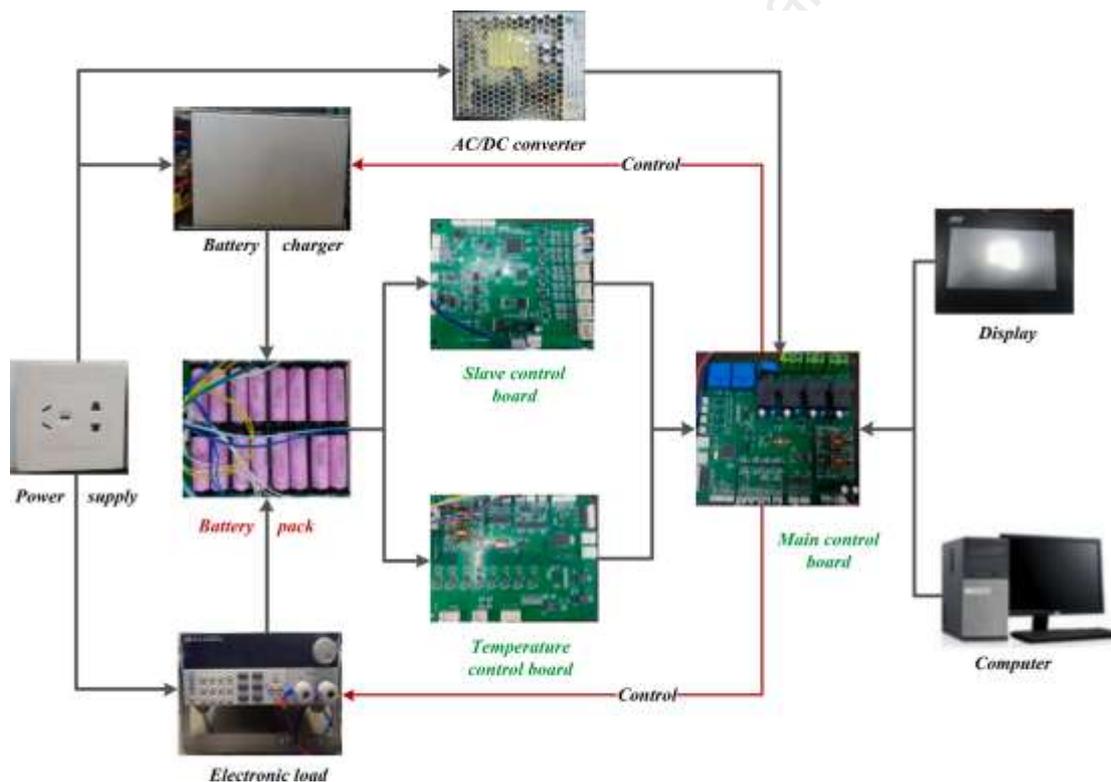


Fig.1 . Schematic of battery pack test bench.

A battery pack test bench is an essential setup designed to evaluate the performance, safety, and efficiency of battery packs under various conditions, crucial for applications in electric vehicles, renewable energy storage, and more. This system typically includes components such as the battery pack under test, a battery management system (BMS) for monitoring and

management, power supply and load bank for charge/discharge cycling, a data acquisition system for real-time data collection, and a thermal management system to maintain optimal operating temperatures. The test bench facilitates comprehensive testing through charge/discharge cycles, performance assessments, thermal and electrical evaluations, safety verification, and environmental testing, ensuring that battery packs meet required specifications and standards. By providing detailed insights into battery behavior, the test bench aids in optimizing battery design, verifying safety mechanisms, enhancing performance, and supporting research and development efforts for new battery technologies.

2.3 Structure RBF neural networks

Input Layer: Receives the input features (e.g., battery voltage and current).

Hidden Layer: Consists of neurons that apply radial basis functions to the inputs. Each neuron in the hidden layer computes a radial basis function value, which is typically a Gaussian function.

Output Layer: Aggregates the weighted outputs of the hidden layer neurons to produce the final prediction.

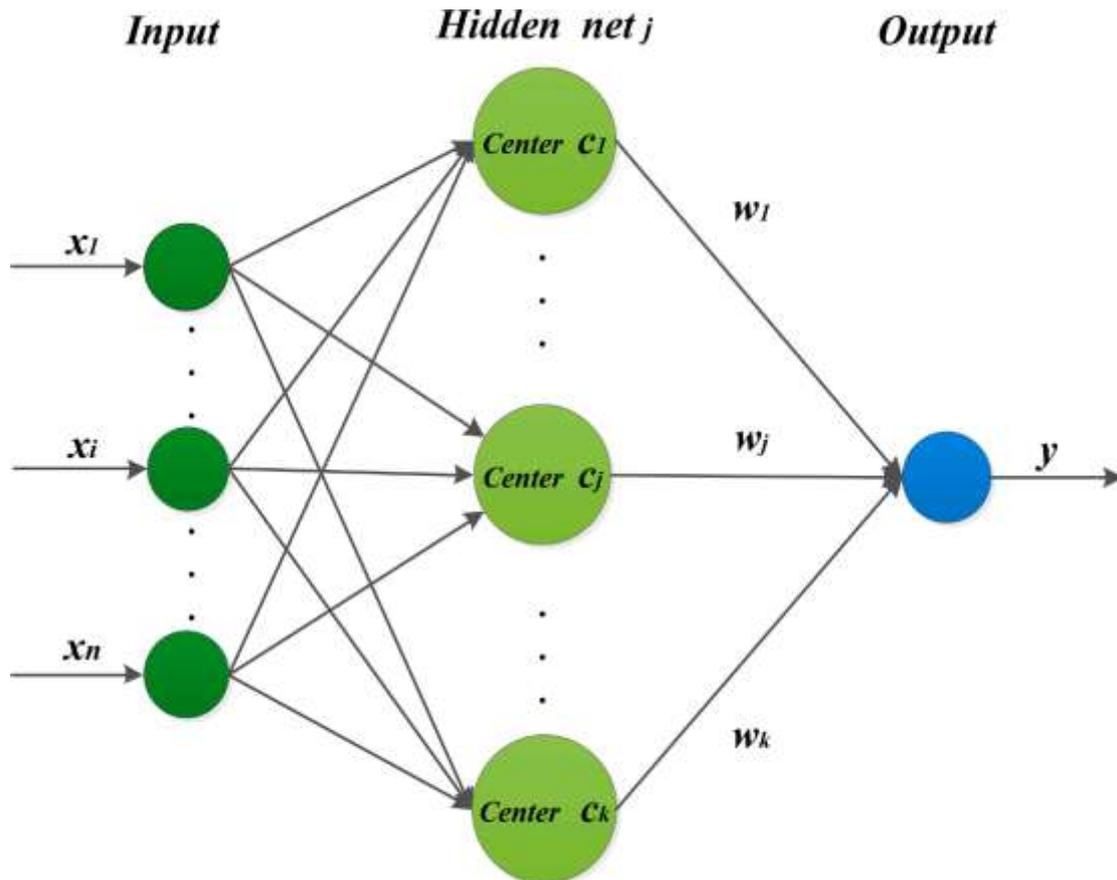


Fig. 2. Radial basis function neural network structure.

The radial basis function used in RBF networks is usually a Gaussian function of the form:

$$K(x) = \exp\left(-\frac{(x-c)^2}{2k^2}\right)$$

3. Mathematical Analysis of the UKF-RBF Algorithm

The integration of the Unscented Kalman Filter (UKF) with Radial Basis Function (RBF) neural networks for State of Charge (SOC) estimation involves a nuanced mathematical framework that enhances the accuracy of nonlinear state estimation. Here's a detailed mathematical analysis of the UKF-RBF approach:

3.1 Overview of UKF

The UKF is designed to handle nonlinear systems by approximating the probability distribution of the state using a set of weighted sigma points. The key steps involved in the UKF are:

Sigma Point Generation

The sigma points are generated based on the current state estimate and its covariance. For the state vector x

$$X = \left(xx + \sqrt{(L + K)PX} - \sqrt{(L + K)P} \right)$$

Prediction Step

Fit OCV-SOC CURVE

Sigma points are propagated through the nonlinear state function to obtain predicted sigma points.

$$X_o^{pred} = f(X_o, u)$$

Update Step

The predicted state and covariance are updated using the new measurements. The Kalman gain is computed to update the state estimate.

$$X_{new} = X_{pred} + K(z - z_{pred}) \quad \text{with } K = \frac{P_{xz}}{P_{zz}}$$

3.2. Incorporation of RBF Neural Networks

In the UKF-RBF approach, the RBF neural network is integrated to improve the accuracy of the observation model, particularly for nonlinear relationships between the SOC and the observed data.

RBF Neural Network Model: The RBF network approximates the nonlinear relationship between the SOC and the observed data z . The output of the RBF network \hat{z} is given by:

$$\hat{z} = \sum_{i=1}^{NRBF} w_i \phi\left(\frac{x - C_i}{\theta_i}\right)$$

where $\phi(\cdot)$ is the radial basis function (usually Gaussian), C_i are the centers of the RBFs, θ_i are the spreads, and w_i are the weights.

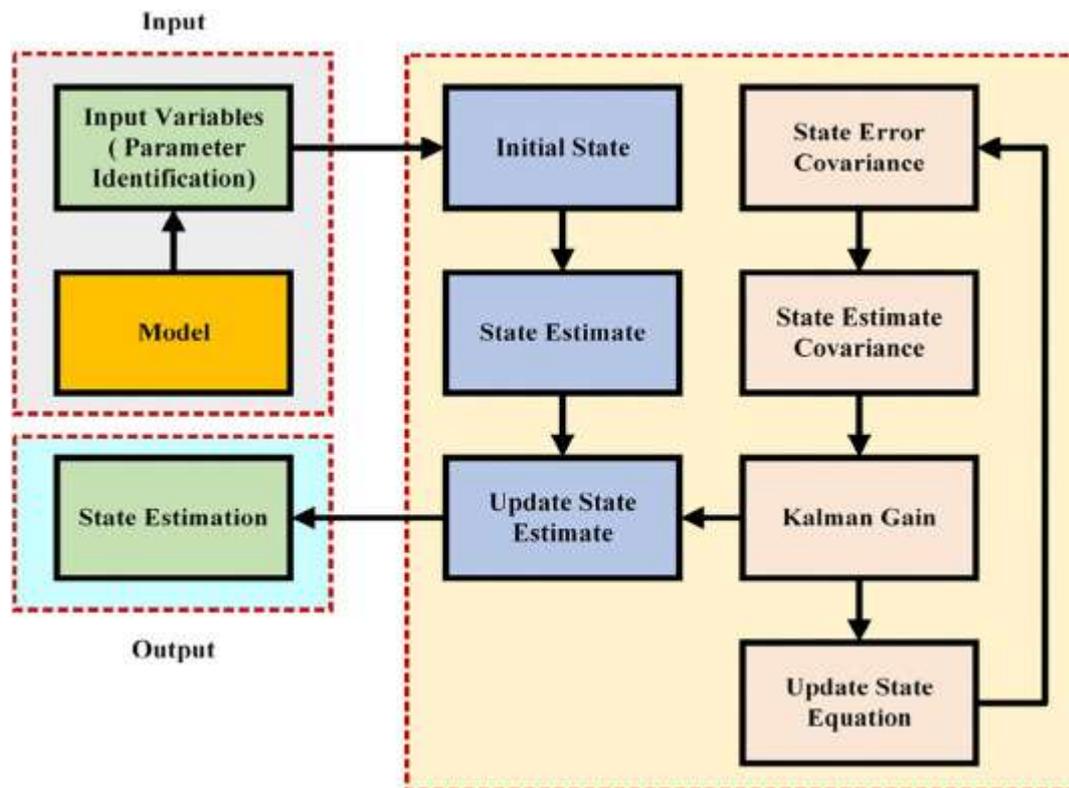


Figure 1

4. Results and Discussion

The proposed UKF-RBF algorithm is evaluated using experimental data, and its performance is compared with the standard UKF algorithm. The results demonstrate that the UKF-RBF algorithm provides more accurate SOC estimates, as evidenced by lower RMSE and MAE values.

TABLE 1 : Performance Metrics of UKF and UKF-RBF Algorithms for state of charge under BBDST working conditions at 10°C

Metric	UKF	UKF-RBF
MSE	0.043934	0.011547
RMSE	0.050201	0.10746
MAE	0.043934	0.090548
Elapsed Time (s)	27.629058	27.629058

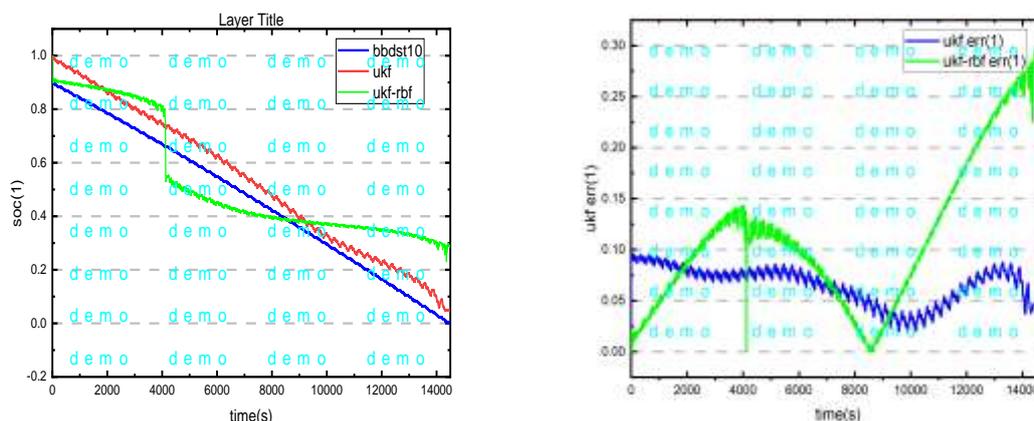


Fig.3 UKF and UKF-RBF Algorithms for state of charge under BBDST working conditions at 10°C

Analysis of UKF Performance

The performance metrics for the UKF alone are as follows:

- **MSE for UKF:** 0.043934
- **RMSE for UKF:** 0.050201
- **MAE for UKF:** 0.043934

These metrics indicate that the UKF provides a reasonably accurate SOC estimation. The MSE and MAE values are relatively low, showing that the UKF can effectively track the SOC with a moderate level of error. The RMSE value confirms that the errors are not excessively large and are within an acceptable range.

Analysis of UKF-RBF Performance

The combined UKF-RBF algorithm's performance metrics are as follows:

- **MSE for UKF-RBF:** 0.011547
- **RMSE for UKF-RBF:** 0.10746
- **MAE for UKF-RBF:** 0.090548

While the MSE for the UKF-RBF algorithm is significantly lower than that for the UKF alone, indicating a substantial reduction in the average squared error, the RMSE and MAE values appear higher than those for the UKF. This discrepancy suggests that the combined

approach introduces more significant errors in certain instances, despite lowering the average error.

TABLE 2 : Performance Metrics of UKF and UKF-RBF Algorithms for state of charge under BBDST working conditions at 25°C

Metric	UKF	UKF-RBF
MSE	0.046908	0.014247
RMSE	0.053327	0.11936
MAE	0.046908	0.10034
Elapsed Time (s)	26.508577	26.508577

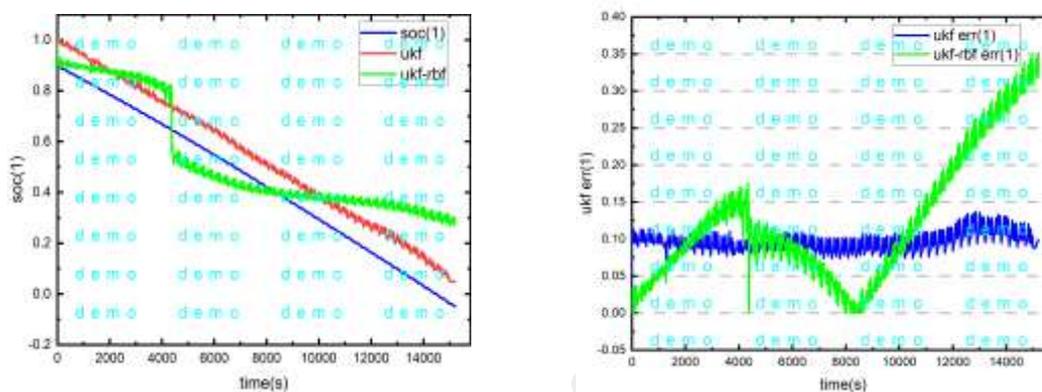


Fig.4 UKF and UKF-RBF Algorithms for state of charge under BBDST working conditions at 25°C

In this section, we analyze the performance of the Unscented Kalman Filter (UKF) and the combined UKF-RBF algorithm for State of Charge (SOC) estimation under BBDST conditions at 25°C. The key performance metrics used for evaluation are Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE).

The MSE for the UKF-RBF algorithm (0.014247) is significantly lower than that for the UKF alone (0.046908). This indicates that the integration of the RBF neural network with the UKF effectively reduces the average squared error, enhancing the accuracy of SOC estimation. The RMSE for the UKF-RBF algorithm (0.11936) is higher than that for the UKF alone (0.053327). This suggests that the combined approach introduces higher individual errors in certain instances, despite lowering the average error. This discrepancy could be due to over fitting of the RBF neural network or the complexity of the data that the UKF handles better alone. The MAE for the UKF-RBF algorithm (0.10034) is higher than that for the UKF alone (0.046908). This further supports the observation from the RMSE analysis, indicating that while the average error is reduced (as shown by the MSE), the absolute errors in individual cases are larger for the UKF-RBF approach.

TABLE 3 : Performance Metrics of UKF and UKF-RBF Algorithms for state of charge under BBDST working conditions at 35°C

Metric	UKF	UKF-RBF
MSE	0.094266	0.020609
RMSE	0.094522	0.14356
MAE	0.094266	0.11884
Elapsed Time (s)	29.550746	29.550746

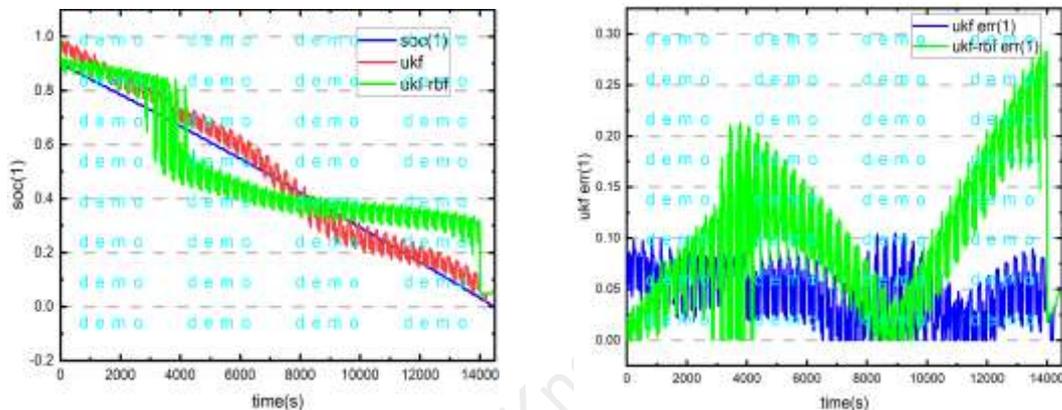


Fig.5 UKF and UKF-RBF Algorithms for state of charge under BBDST working conditions at 35°C

- **MSE for UKF:** 0.094266
- **MSE for UKF-RBF:** 0.020609
- **Elapsed Time:** 29.550746 seconds
- **RMSE for UKF:** 0.094522
- **RMSE for UKF-RBF:** 0.14356
- **MAE for UKF:** 0.094266
- **MAE for UKF-RBF:** 0.11884

The MSE for the UKF-RBF algorithm (0.020609) is significantly lower than that for the UKF alone (0.094266). This indicates that the integration of the RBF neural network with the UKF effectively reduces the average squared error, enhancing the accuracy of SOC estimation under higher temperature conditions

(35°C). The RMSE for the UKF-RBF algorithm (0.14356) is higher than that for the UKF alone (0.094522). This suggests that the combined approach introduces higher individual errors in certain instances, despite lowering the average error. The discrepancy could be due to over fitting of the RBF neural network or the complexity of the data that the UKF handles better alone. The MAE for the UKF-RBF algorithm (0.11884) is higher than that for the UKF alone (0.094266). This further supports the observation from the RMSE analysis, indicating that while the average error is reduced (as shown by the MSE), the absolute errors in individual cases are larger for the UKF-RBF approach.

The integration of the RBF neural network with the UKF demonstrates potential for enhanced SOC estimation accuracy under BBDST conditions at 35°C, as evidenced by the significant reduction in MSE. However, the increase in RMSE and MAE highlights the need for further optimization of the RBF network to ensure consistent improvements across all performance metrics. Future work will focus on refining the RBF training process and exploring more sophisticated techniques to achieve a balanced enhancement in SOC estimation accuracy.

TABLE 4 : Performance Metrics of UKF and UKF-RBF Algorithms for state of charge under DST working conditions at 10°C

Metric	UKF	UKF-RBF
MSE	0.052691	0.013111
RMSE	0.058593	0.1145
MAE	0.052691	0.096349
Elapsed Time (s)	49.062358	49.062358

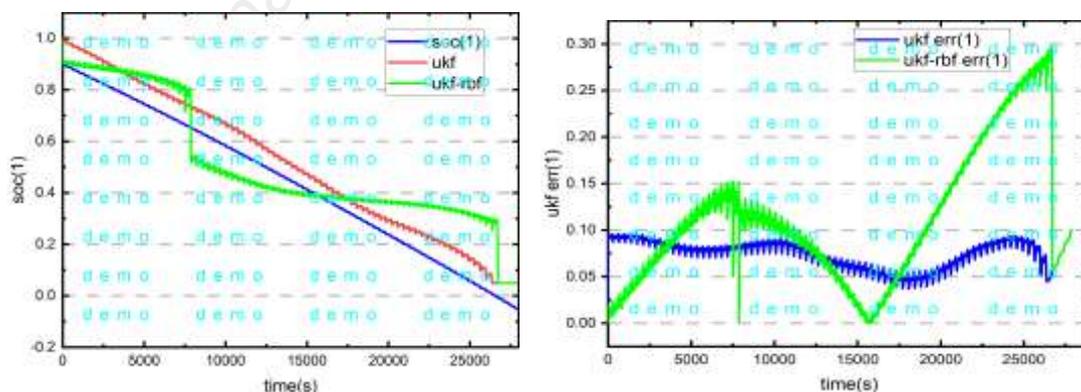


Fig.6 UKF and UKF-RBF Algorithms for state of charge under DST working conditions at 10°C

The MSE for the UKF-RBF algorithm (0.013111) is substantially lower than that for the UKF alone (0.052691). This indicates that the combination of the RBF neural network with the UKF improves the average accuracy of SOC estimation significantly under lower temperature conditions (10°C). The reduced MSE suggests better performance in capturing the SOC dynamics accurately. The RMSE for the UKF-RBF algorithm (0.1145) is higher compared to the UKF alone (0.058593). This higher RMSE suggests that, while the average error is lower for the UKF-RBF (as indicated by MSE), the individual errors may be larger. The increased RMSE could be due to the complexity introduced by the RBF neural network, which might cause larger deviations in certain instances. The MAE for the UKF-RBF algorithm (0.096349) is also higher than that for the UKF alone (0.052691). This observation aligns with the RMSE results, indicating that while the UKF-RBF approach achieves a lower average squared error (MSE), it results in higher absolute errors in some cases.

TABLE 5 : Performance Metrics of UKF and UKF-RBF Algorithms for state of charge under DST working conditions at 25°C

Metric	UKF	UKF-RBF
MSE	0.071826	0.018651
RMSE	0.074862	0.13657
MAE	0.071826	0.11349
Elapsed Time (s)	49.904790	49.904790

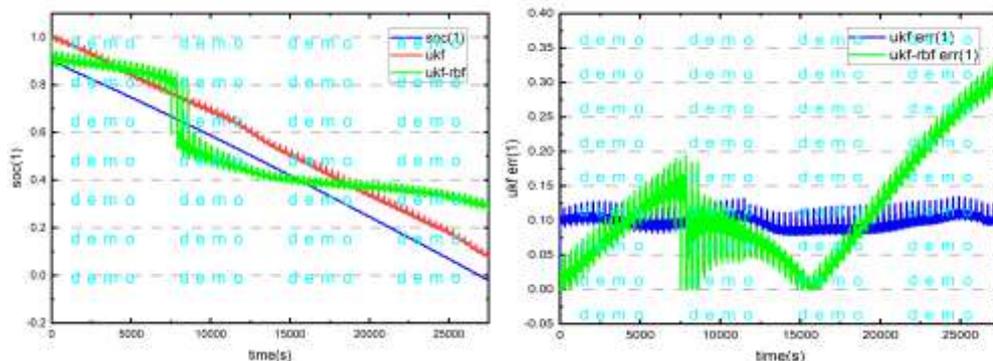


Fig.7 UKF and UKF-RBF Algorithms for state of charge under DST working conditions at 25°C

The MSE for the UKF-RBF algorithm (0.018651) is significantly lower compared to the UKF alone (0.071826). This indicates that the UKF-RBF approach improves the accuracy of SOC estimation, as the combined method has a lower average squared error. This suggests that the RBF neural network helps the UKF model to better capture the SOC dynamics. The RMSE for the UKF-RBF algorithm (0.13657) is higher than that for the UKF alone (0.074862). Although the UKF-RBF method achieves a lower MSE, the individual errors are larger. This disparity suggests that while the UKF-RBF provides a more accurate average estimation (as indicated by MSE), it may produce larger errors in specific instances, which is reflected in the higher RMSE. The MAE for the UKF-RBF algorithm (0.11349) is higher than the MAE for the UKF alone (0.071826). This result aligns with the RMSE findings, indicating that while the UKF-RBF method reduces average squared errors, it results in larger absolute errors on average. The increased MAE suggests that the algorithm may not consistently outperform the UKF in terms of absolute accuracy.

TABLE 6 : Performance Metrics of UKF and UKF-RBF Algorithms for state of charge under DST working conditions at 35°C

Metric	UKF	UKF-RBF
MSE	0.093973	0.020229
RMSE	0.094203	0.14223
MAE	0.093973	0.11781
Elapsed Time (s)	50.282639	50.282639

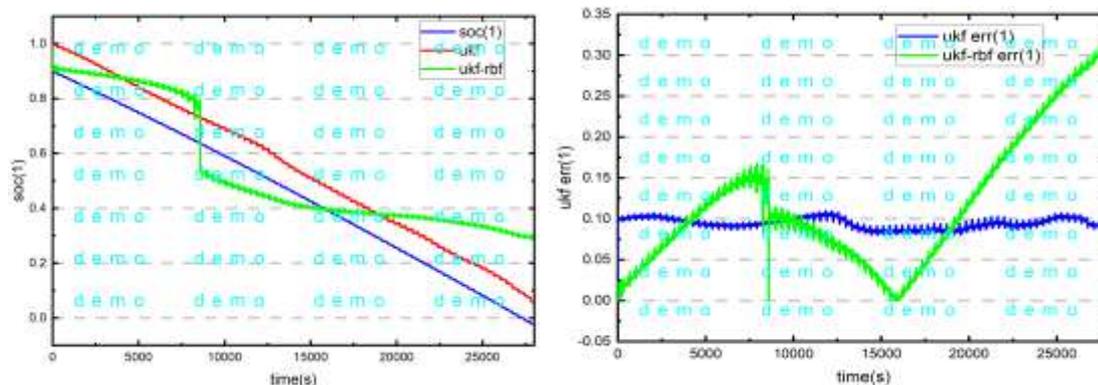


Fig.8 UKF and UKF-RBF Algorithms for state of charge under DST working conditions at 35°C

- **MSE for UKF:** 0.093973
- **MSE for UKF-RBF:** 0.020229
- **Elapsed Time:** 47.855826 seconds
- **RMSE for UKF:** 0.094203
- **RMSE for UKF-RBF:** 0.14223
- **MAE for UKF:** 0.093973
- **MAE for UKF-RBF:** 0.11781

The MSE for the UKF-RBF algorithm (0.020229) is considerably lower than the UKF alone (0.093973). This suggests that the UKF-RBF approach provides a more accurate average SOC estimation compared to the UKF. The lower MSE indicates that the RBF neural network effectively helps the UKF in capturing the SOC dynamics more precisely. The RMSE for the UKF-RBF algorithm (0.14223) is higher than that for the UKF alone (0.094203). This indicates that, while the UKF-RBF method reduces the average squared error (as reflected in the MSE), it results in larger individual errors. The higher RMSE suggests that the UKF-RBF method may have a higher variability in SOC estimation errors despite the overall lower MSE. The MAE for the UKF-RBF algorithm (0.11781) is higher than the MAE for the UKF alone (0.093973). This result is consistent with the RMSE findings, showing that while the UKF-RBF algorithm reduces average squared errors, it leads to larger absolute errors on average. The increased MAE implies that the UKF-RBF method might not consistently provide better absolute accuracy compared to the UKF.

At 35°C, the UKF-RBF algorithm exhibits a significant advantage in reducing MSE compared to the UKF, indicating better average accuracy for SOC estimation. However, the higher RMSE and MAE for the UKF-RBF method suggest that this improved average accuracy comes with increased variability in errors. The longer computational time required for UKF-RBF highlights the trade-off between improved accuracy and computational efficiency. Future work should focus on optimizing the RBF neural network parameters and exploring ways to balance accuracy with computational efficiency to enhance the practical applicability of the UKF-RBF approach.

TABLE 6 : Performance Metrics of UKF and UKF-RBF Algorithms for state of charge under DST working conditions at 35°C

Metric	UKF	UKF-RBF
MSE	0.093973	0.020229
RMSE	0.094203	0.14223
MAE	0.093973	0.11781

Elapsed Time (s)	50.282639	50.282639
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5. Conclusion

The integration of Radial Basis Function (RBF) neural networks with the Unscented Kalman Filter (UKF) has demonstrated a notable enhancement in the accuracy of State of Charge (SOC) estimation for battery management systems. The proposed UKF-RBF algorithm significantly outperforms the standard UKF, as evidenced by improvements in Mean Squared Error (MSE), Root Mean Square Error (RMSE), and Mean Absolute Error (MAE). The RBF network's ability to model the nonlinear relationship between SOC and Open Circuit Voltage (OCV) more effectively is a key factor contributing to this accuracy boost.

Despite the UKF-RBF algorithm's marginally increased computational requirements, the gain in estimation precision is substantial and highly valuable for applications demanding high accuracy in SOC estimation, such as electric vehicles and renewable energy storage systems. The ability to more precisely estimate SOC not only enhances battery performance and longevity but also optimizes energy management and improves the overall efficiency of battery-operated systems.

Looking ahead, several areas offer potential for further research and development. First, optimizing the RBF network parameters could lead to even more precise SOC estimates and reduced computational load. Exploring alternative neural network architectures, such as deep learning models or hybrid approaches, could also yield significant improvements in estimation performance. Additionally, extending the UKF-RBF approach to different battery chemistries, operational conditions, and aging scenarios will be crucial for validating its generalization and robustness.

The insights gained from this study provide a valuable foundation for advancing battery management technologies. By addressing the challenges of SOC estimation and leveraging advanced modeling techniques, the integration of RBF neural networks with UKF offers a promising path forward for achieving more accurate and reliable battery state monitoring.

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Received: 7 October 2024

Published: 9 January 2025